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DISCOVERING THE META- CENTRIC HEIGHT OF VESSEL IN A MORE RATIONAL WAY, BY ANGLE OF OSCILLATION & DOING ITS INVESTIGATION WITH REGGARDS OF STABILITY ALONG WITH DIMENSIONAL ANALYSIS

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ABSTRACT

Height of the meta-centre of vessel is the basis of discussion in this study which is kept as conventional for its methodology as it should be, except the modification of the discussion. This study is regarding discussion of the methodology which describes the approach to find the Meta-centric (GM) height, i.e. the meta-centric distance of vessel. The vessel when tilted has been discussed with more precise estimation with its entire physical inclusion. Variety in vessel configuration is a common instance & but for all how a generalized form of the GM can be expressed so precisely so that it can reflect the economy by engineering in choosing/selecting a vessel of a particular configuration has been broadly described & this mode/the finding about the selection has achieved a new dimension by the innovation as obtained by this study. Besides, the energy that can be derived out & developed using the GM equation has been discussed with. Summarily, the geometrical design of the vessel, by the selection, must find its some ways or options to find out the most suitable one satisfying the various background challenging criteria. This study has visualised the formulation of the meta-centric height as derived/obtained through its related possible evaluation which besides showing the way of comparison amongst gives several useful outlets also along-with its future possibility, by the vessel size, mode of GM formation & stability.

Keywords: Vessel, Centre of buoyancy, Tilt, Meta-centric distance, Moment of inertia, Dimensional Analysis, Stable vessel.

NOTATION: Here is the brief description of the symbol used in this study.

P = Centre of Buoyancy
G = Centre of Gravity
M = Meta-centre
I = Moment of Inertia of vessel
¥ = Volume of the Water Displaced by Vessel
θ = Angle of Tilt
° = deg. = Degree (Unit of Tilt)
i.e. = that is
* = Multiplication symbol

1. INTRODUCTION

Limitations demarcate everything to abide by. Also, the demand on excess or more

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exists suitably. Regarding vessel on water carrying the various commercials suffers from getting the more spaces, on every aspect. Vessel on water has although its own way of movement based on various impounding factors responsible for the limitation in its aspects. The limitation creates the boundary of do's & don'ts. The degree of oscillation with which vessel goes through decides the fate of the vessel ultimately. The carrying capacity of a vessel with adequate safe configuration is very important from commercial view-point. So far, there are lots of the vessels made with various changing patterns in shape, size & the capacity. Till so far whatever the progress about the meta-centric height is, the governing design value is getting implemented varyingly for the different vessels subjectively – the type & nature of loading, geometrical configuration& the pattern of shipping (particularly, the length of voyaging) etc. must be the factors behind. The basic fundamental of vessel design is on basic physical features of vessel like Center of Buoyancy(P), Center of Gravity(G), Meta-center(M), the wave-based desirable fulfillments, etc. – the dynamic property of these factors are to be used considerably for making a vessel proper & suitable. The interconnecting distance between respective any two points (P, G or M) gives the particular knowledge in knowing the behavior of any particle floating on water or else. Each length such as PG, GM & PM of vessel denotes behavior in water even during rest also (Fig.1a), condition of G of vessel (Fig.1b) with respect to M & P (by formation of M) respectively. The entire contribution of each of these three lengths or distances of vessel is ultimately exhibited by the formation of Meta-centric height with which vessel is used to go by. The length GM is popularly known & called as Meta-centric height.

As like PG, PM is also associated with GM, particularly into the mathematical evaluation. Stability & instability of vessel is on the factor of how GM height goes to gaining the magnitude of severity – the loss & gain in the value of GM defines the vessel's stability. Everywhere in vessel, the only objective to keep the vessel under stable vision of its driving has been the satisfactory abidance on the statutory governing condition i.e., PG < GM, on all the time of the sailing [Bansal. 2002]. Vessel's geometric design often struggles with the basic equation of buoyancy, i.e. $GM = \left(\frac{I}{¥}\right) - PG$ for attaining at the meta-centric height of some desired ones in order to bring the suitable form of itself into reality; where, I = Moment of Inertia of vessel. ¥ =Volume of water displaced

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

by vessel. This basic equation of GM has been getting recognized as the conventional one in the subject of buoyancy.

It's always desired to beautify engineering excellences. The human civilization, so far of today, is cherishing this beauty. This attempt is not the new one; rather it's been since human's ability open fire, possibly. It's so eye-charming to see vessels floating with their hugely structures as well as its calibers on sea by following the fundamental rule of buoyancy, particularly of the governing lengths. With suitable variation on the variables involved in GM height, the large, voluminous-type look or the castles are built abiding applicable stability concern given by the GM equation. The fundamental equation of PM is defined as,$PM = \left(\frac{I}{¥}\right)$; where, PM = the factor or length governing to forming the GM. With this general formula of PM the distance GM is determined from the physical position (of points) of vessel itself & subsequently it's used in the suitable design of vessel. The line on which the physical points M, G & P lie (under certain tilt) may be regarded as control-line (M-G-P) of vessel. Along this control-line, there's always a continuous play gone through by the points M, G & P. During the oscillation only, the plays of the points (M, G & P) along the line M-G-Pare by nature of the driving elements or waves so created are the matter of degree of stability [Wikipedia, GM. 2018]. For information, the conventional 'GM' value for most of the vessel is kept in the range from 0.3m to 1.2m. For warships, it is of 1m to 1.5m [Bansal. 2002]. The struggle in obtaining stability is thereby on towards getting vessels on its physical stability concern during oscillation of the meta-centric height or the travel of the point 'M' along the so-called M-G-P (Fig.1) with the simultaneous fulfillments of the desired vessel [NSTM].

Formation of Meta-center (M) is by law of nature for vessels. The magnitude of GM expresses the amount of tilt (angular displacement) given to the vessel. This study has described a theoretical methodology by which the 'conventional' GM height has got its new broader dimension. The basic aim has been given on seeing through the effects on GM by varying natures of the value of the angle of inclination (i.e., tilt) & position of 'P' while deriving out the equation of GM. There are various conditional equations of GM found which are self-explanatory by their own way of formation. The theoretical interests have been given on to the deriving the equation subjectively &

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this attempt to find out all the possible situations of a vessel on floating water may also be in assessing the effects of the various GM formulations on the stability of vessel [Mégel & Kliava .2015]. There should not be thereby any best condition to choose from the given or discussed ones possibly. Instead, the condition which might be created 'artificially' by combating various challenging forces in order to establish a particular type of the vessel that may be required or happened to be innovated for use by objectives or for requirements of certain situation. The conditions (the 8 Cases) are all encompassing on the value of the tilt & based on the behaviors of the center of buoyancy (P) subsequently. In doing so, assumptions have been expressed in connection with these. These conditions are, in this study, termed as 'Case'. The incorporation of the tilt(θ) & position of 'P' into the meta-centric height as determined the more insight-views of the various findings associated with the GM height. Now the derived equations of GM could show detailing of meta-centric phenomenon. These GM formulations may be the substitute of the conventional GM of vessel. The conventional GM may also get its comparison by the GM equations so derived herein. It describes on how the overall configuration of vessel can possibly be able to get changed, even for getting its desired capacity indeed, with sufficient modes of stability. Further, critical limitation in the tilt value for the GM has been shown graphically, although this limitation may also be done by the varying position of 'P' indeed. With this, future scopes are existing on all through in this study & that's quite nonetheless.

2. GOALS OF STUDY

Followings are the goals & aims of this theoretical study –

a) To evaluate & determine the meta-centric height of vessel with angular aspects.

b) To find & incorporate the GM by the suitable variation in angle MPP_1 or MP_1P & the angle FNQ by mode of selective desiredness (Fig.1 & Fig.2).

c) To have a look out with the as-usual conventional equation of GM.

d) To determine degree of easeness of the GM equation by physicality so far on desired basis.

e) To have a view & place of this study's proposition towards its possible furtherance of future scope.

3. ASSUMPTION

Followings are the assumption underlying this study –

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

i) The medium of vessel's floating is on water.

ii) Vessel's configuration shown in figures is exemplary & not to scale.

iii) The methodology, entirely, follows the basic general rules of mechanics & fluid mechanics.

iv) The angle FNQ is assumed as the angle of tilt (= θ = angle PMP_1) in all aspects (Fig.1 & Fig.2).

v) The vertical component (W) of the line of action of the strip-weights of vessel is parallel & similar in direction to the total weight of the vessel (Wv) through the center of gravity (G) of vessel; segmental distances of the strips are considered with respect to these vertical components perpendicular to the Y-Y axis.

vi) Mode of the angles as considered by combination as said in (iv) & (v), above, is subjectively varying (Fig.4).

4. METHODOLOGY

The methodology described here in this paper has narrated & evaluated a theoretical methodology extensively, in obtaining the meta-centric height of vessel. In this study, a schematic vessel is considered for the entire delineation of the GM evaluation. The vessel which is shown in the Fig.1 & Fig.2 with whom the related evaluation as described in this study is equally applicable for the mechanism/methodology stated rather than the feature or physical scalar dimension of the vessel itself as it may be applied decidedly, indeed, as it's the stage of theoretical & dimensional justification prior to the implementation. The methodology is on the **inclusion of the angular provision** as (to be) formed in the vessel's physicality. The methodology at first has discussed the formulation of the related equation of PM in 'general form' [Wikipedia, Buoyancy. 2018], as a function of the tilt whose case-wise forms have been obtained afterwards. Correspondingly, all the PM equation is having the particularity in their forms of the cases specified. Self-expressive matters have been ignored, except only the essentials aiming at the goals of this study whose derivation as well as determination is based on the similar 'conventional' way of formulation of GM height, to some extent – the specialty of the angular aspect is thereby the issue here & its basis of formulation is given in the assumption in detail. By the way, the theoretical approach or the mathematical methodology is described as follows –

Description on the Derivation of Meta-centric Height (GM)

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

The Fig.1 shows the plan & elevation of a vessel on water medium, both in stationery (Fig.1a) & in tilted condition (Fig.1b). The vessel is symmetric about Y-Y axis. The Sectional contribution of the areas of the wedge of the vessel after given the angular displacement (tilt) as shown in the Fig.1c is the wedge useful in deriving the GM height finally.

In order to derive the GM height, vessel is to be given the angular rotation or displacement which is called as tilt(θ). Let, b = total width of vessel at the water line (Plan-view); L = length of vessel or vessel at the surface of the free-water line (in Plan-view); d = depth of submergence of vessel with respect to the free-water surface.

After the angular displacement(θ)is given to vessel, the Meta-centric point (M) occurs & consequently the vessel starts oscillating in a pattern like as shown in the Fig.1b & Fig.2. The Meta-centric height (GM) determination is the objective of this study. In this study it is again assumingly said that the small inclination(θ) also results to the same magnitude of angle at the point 'N' of vessel, by forming the wedge-shaped strips of the inclined vessel-surface with respect to the fixed water-line. The point N is the junction point of the surface of water-line & vessel.

Considering a strip of wedge of thickness 'dx' at a distance 'x' from the axis Y-Y (Fig.1a), the determination of the GM height is done as followed –

Let, the angle DNR of the wedge NFQ = angle DNR = angleD_1NR_1 = θ degree = the tilt = angle PMP_1

In this study, the entire wedge area is divided into the one as shown in the Fig.1 where it's shown that the wedge area is considered to be constituting of the trapezoidal section & triangular section; the section giving the triangular geometry in wedge lies only at the corner of the wedge NFQ – this distribution in the geometry of the wedge NFQ is symmetric about the Y-Y axis by the entirety of the vessel. In the determination of the wedge-area, the value of (θ) may be in the form of sin(θ) or,tan(θ), depending on the magnitude of (θ) itself, the geometry of vessel, degree of rotation applied, etc. & this varying effects of trigonometry in the value of (θ) are functionally attached to the angle of (θ) itself – this is one of the assuming criteria in forming the eight cases (discussed afterwards; Table 1 to 3) in the GM height determination of this study.

Centroidal distance of each (segmental) trapezoidal wedge,

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$$x_{wg} = \left[\frac{x\theta + 2(x+dx)\theta}{(x+dx)\theta + x\theta}\right]\left(\frac{dx}{3}\right)$$

$$= \left(\frac{1}{3}\right)\left\{\frac{3x + 2(dx)}{2\left(\frac{x}{dx}\right) + 1}\right\}$$

The centroidal distance (from point N) of the triangular wedge considered here is much lesser than the positioning of x_w by value, of the wedge NFQ defining in for the determination of the meta-centric height; thereby, it's neglected.

Centroidal distance of the wedge NFQ, x_w from the point N is given by,

$$(x_w) = (x_{wg} + x)$$

$$= \left(\frac{1}{3}\right)\left[\frac{6x + 6\left(\frac{x^2}{dx}\right) + 2(dx)}{2\left(\frac{x}{dx}\right) + 1}\right]$$

Now from Fig.1 & Fig.2, the weight of the wedge-area is to be estimated. In this regard, the angular contribution by each linear side of the wedge NFQ has been taken into the determination, so far as the geometrical distribution, by dimension of wedge, is concerned, as said earlier. Area of the trapezoidal wedge is given by,

$(a_p) = [x\theta + (x+dx)\theta]\left(\frac{dx}{2}\right) = [2x + (dx)]\left(\frac{\theta}{2}\right)dx = [2x(dx) + (dx)(dx)]\left(\frac{\theta}{2}\right)$; where $\theta = \tan\theta = \sin\theta$.

Here it is required to be mentioned that this consideration of θ, here, is completely case-specific as discussed in itself in the wedge NFQ. In the use of this θ, there may be some inclusion by its angular dimension (i.e., by tangent o sine or cosine aspect) to cases as required as described in the subsequent formation in the GM height enunciation. By doing that, corresponding derivations may be obtained.

By the way, in order to consider the triangular wedge into the estimation of the wedge area, a factor is here considered which is equal to $(f) = (a_r/a_p)$; $0 < f < 1$; where, a_r = area of triangular wedge. This factor 'f' has made the equation into quite simplified & also determinative. As clearly shown in the Fig.1, the inclusion of the geometrical distribution of wedge into trapezoidal & triangular is of theoretical interests & also for the requirement of this study, indeed, even to the extent of its importance also. Thereby, the area of the entire (segmental) wedge (a_w) = Area of the (trapezoidal + triangular) wedge. **(Ref Figure – 1 & 2)**

Thereby, $(a_w) = (a_p) + (a_r) = (a_p) + f(a_p) = \{1 + f\}(a_p) = (1+f)[2x(dx) + (dx)(dx)]\left(\frac{\theta}{2}\right)$

Let, $\left[\frac{(a_p)}{(a_p)+(a_r)}\right]100 = P_p =$ Contribution of the trapezoidal wedge(s) in percentage. This value of P_p may be merged with the

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

triangular wedge(s) in order to make it into a singular geometrical shape (trapezoidal) of determination, as a whole, of the vessel. Or, the reverse of this transformation may also be done into the further calculation of the GM determination, by making the entirety into the singular triangular shape by geometry, instead of the trapezoidal one. This percentage distribution (over P_p) shall give advantageous sides in the design of vessel as described in this study.

Now, the factorial term (1+f) is equal to $(1+f) = \left[\frac{(a_p)+(a_r)}{(a_p)}\right] = \left(\frac{100}{P_p}\right)$

Thereby the equation of the total wedge-area (a_w) is given by, $(a_w) = \left(\frac{100}{P_p}\right)[2x(dx) + (dx)(dx)]\left(\frac{\theta}{2}\right)$

Volume of the wedge NFQ, V = (wedge-area) Length

$$V = (a_w)(L_{eq.})$$

$$= \left(\frac{100}{P_p}\right)[2x(dx) + (dx)(dx)](L_{eq.})\left(\frac{\theta}{2}\right) \qquad \text{... (v1)}$$

where, $(L_{eq.})$ = equivalent length of vessel. The value of this $(L_{eq.})$ or the volume V is to be the basis of research interests as well as from the practical point of view as may be considered for the geometrical configuration in the vessel dimension. Entire description of this study has been given by using the Eq.(v1) so far as the vessel configuration (Fig.1a) is given concerned of to bring out the GM determination. The variable 'L' which is shown in the Fig.1a is the length of vessel along a particular side of the vessel. It may change while the vessel would be of irregular pattern or rather consisting different plan-area along its width (b), that means when the vessel's plan view is having with the different lengths across its width (b), then the equivalent concept shall come into picture. And, this study has encouraged in its derivation the irregular pattern in the plan-view of vessel. So, for such vessel, the length has been taken here by an equivalent measure & it is the $L_{eq.}$which would be taken here in all the applicability in terms of the length criteria to be fulfilled for the vessel's length consideration. In the foregoing discussion it might be realized to see the use of this $L_{eq.}$in the way it's getting implemented as just a replacement of the L, but it is not like that; the variable $L_{eq.}$ whenever be used & found, it should be considered & visualized it for a vessel having different lengths along its total width(b). It is not equal to L at all, rather than it's better to be related with the average or weighted average kind of estimation over the combinations of various L's & b's of

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

vessel. To be simple, $L_{eq.}$ is thereby going to be placed in place of L, for keeping its effect as simple as for the derivation of the Meta-centric height which is the cardinal objective of this study.

Although, the Eq. (v1) may also be written for the 'entire' vessel having irregular geometry of dimensional suitability as per the following one,

$$V = \sum\{(a_w)(L_{eq.})\}$$

$$= \sum\left\{\left(\frac{100}{P_p}\right)[2x(dx)\right.$$

$$\left.+ (dx)(dx)](L_{eq.})\left(\frac{\theta}{2}\right)\right\} \qquad \ldots (v2)$$

Eq. (v1) is the general equation of the strip-volume in terms of the angle (θ)& particular configuration as well. Weight of the wedge= $W = (\rho g)(V)$; where, ρ = density of water; g = acceleration due to gravity.

The weights (W) of strips of wedges on both sides of the dividing axis Y-Y are acting vertically downwards & vertical component of its line of action is similar in direction & parallel to the total weight of the vessel acting through the C.G, center of gravity of the vessel. These strip-weights on both sides of Y-Y axis do finally make a couple which acts in opposite direction to the given rotation, on the vessel on a whole. The force of this resisting moment is the weight of the wedge NFQ which is passing through the center of gravity (Z) of wedge & it's parallel to the (Wv) of vessel (Fig.1) – it is one of the basic background guideline of this study. The resisting moment is here termed as 'the wedge-moment' as offered by the wedge of vessel.

The defining distance of the wedge-moment of the wedge NFQ from the point N is the distance which is produced on both sides of vessel is the distance required for the creating the resisting moment against the applied rotation in opposite direction. Thereby, distance between the C.G of the wedge for the entire vessel (on the both side) = the distance between the vertical components of the weights of such two strips of the wedge is= $(x_w + x_w) = 2(x_w) = \left(\frac{2}{3}\right)\left[\frac{6x+6\left(\frac{x^2}{dx}\right)+2dx}{2\left(\frac{x}{dx}\right)+1}\right]$. This is the total distance of the force of the wedge-moment caused by the given rotation.

Now, Total wedge-moment = sum of the moments caused by the weights of the strips of the wedge = ∑ (Force multiplied by the (perpendicular) distance in between) = $(M_w) = \sum(W\cos\theta)(2x_w)$

- ➢ $(M_w) = \sum(\rho g)(V)\cos\theta(2x_w)$

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

- $(M_w) =$

$$\left(\frac{1}{3}\right)\oint(\rho g)\left(\frac{100}{P_p}\right)\left[\frac{6x+6\left(\frac{x^2}{dx}\right)+2dx}{2\left(\frac{x}{dx}\right)+1}\right][2x(dx)+(dx)(dx)](L_{eq.})(\theta)\cos\theta$$

The term $\left[\frac{6x+6\left(\frac{x^2}{dx}\right)+2dx}{2\left(\frac{x}{dx}\right)+1}\right][2x(dx)+(dx)(dx)]$ is evaluated as followed –

$$\left[\frac{6x+6\left(\frac{x^2}{dx}\right)+2dx}{2\left(\frac{x}{dx}\right)+1}\right][2x(dx)+(dx)(dx)]$$

$$=\left[\frac{6x+6\left(\frac{x^2}{dx}\right)+2dx}{(2x+dx)dx}\right](dx)[2x+(dx)]dx$$

$$=\left[6x+6\left(\frac{x^2}{dx}\right)+2dx\right]dx$$

$$=\left[6(x^2dx)\left(\frac{1}{dx}\right)+6(xdx)+2(dx)(dx)\right]$$

$$(M_w)=\sum(\rho g)(V)\cos\theta(2x_w)$$

$$=\left(\frac{1}{3}\right)\oint\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[6(x^2dx)\left(\frac{1}{dx}\right)+6(xdx)+2(dx)(dx)\right](L_{eq.})(\theta)\cos\theta \qquad \text{... Eq. (i)}$$

Eq. (i) is the final general form of the wedge-moment (total) in terms of the involved unique variable.

Method of Evaluation of M_w:

Now the Eq. (i) of M_w is evaluated by the following mathematical approach using direct & indirect aspect of its own definition which is regarded here as Direct & Indirect method. Both the methods simplify the equation of M_w of Eq. (i) in the way for which it's been classified so.

The limit of the integration of the integral terms of Eq. (i) is to be from zero to (x_w) to so far as the (defined) wedge (NFQ), as a whole, is considered here for the calculation purpose. The extent of (x_w) is from point N to point F/Q of the wedge NFQ (Fig.1). As each term's integration is given here for the upper limit of $(x_w) \approx \left(\frac{b}{2}\right)$ & the lower limit as zero, so far as it's relation is attached to with the vessel (Fig.1 & Fig.2), the required evaluation of the integration involved in Eq.(i) is done as followed –

$$\oint dx = 2\int_{x=0}^{(x_w)\approx\left(\frac{b}{2}\right)} dx = b$$

$$\oint xdx = 2\int_{x=0}^{(x_w)\approx\left(\frac{b}{2}\right)} xdx = 2\int_{x=0}^{\left(\frac{b}{2}\right)} xdx$$

$$=\left[\frac{b}{2}\right]^2=\frac{b^2}{4}$$

$$\oint x^2dx = 2\int_{x=0}^{(x_w)\approx\left(\frac{b}{2}\right)} x^2dx = \left(\frac{2}{3}\right)\left[\frac{b}{2}\right]^3$$

$$=\frac{b^3}{12}$$

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$I = \oint 2x^2 dA =$ Second Moment of Area of the vessel under buoyant action at the free-water surface about the Y-Y axis. And, for the entire vessel, the total moment of inertia= (2I).

Thereby, the required integral values are determined as shown in the following:

$$\oint\left(\frac{x}{dx}\right) = 2\int_0^{b/2}\left(\frac{1}{dx/x}\right) = \left[\frac{2}{\log_e\left(\frac{b}{2}\right)}\right]$$

$$\oint x^{-2}dx = 2\int_{x=0}^{(x_w)\approx\left(\frac{b}{2}\right)} x^{-2}dx$$

$$= (-2)\,(b)^{-1} = (-)\left(\frac{2}{b}\right)$$

Now the evaluative methods are described as the direct & indirect method wherein the above values of the integrals have been applied suitably.

Direct Method:

It is the approach by which the formulation of M_w of Eq. (i) is determined by estimating the general integration as be applied for its subsequent required derivation. In this method, the term $\left[6(x^2dx)\left(\frac{1}{dx}\right) + 6(xdx) + 2(dx)(dx)\right]$of Eq. (i) is evaluated here as followed –

$$\oint\left[6(x^2dx)\left(\frac{1}{dx}\right) + 6(xdx) + 2(dx)(dx)\right]$$

$$= 6\left(\frac{b^3}{12}\right)\left(\frac{1}{b}\right) + 6\left(\frac{b^2}{4}\right)$$

$$+ 2b^2 = \left(\frac{b^2}{2}\right) + \left(\frac{7}{2}\right)b^2$$

$$= 4b^2$$

From Eq. (i), $(M_w) = \left(\frac{1}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(L_{eq.})(4b^2)(\theta)\cos\theta$

$$(M_w) = \left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\theta)\cos\theta$$

where, $A = \sum(L_{eq.})\,(dx) = (b)\sum(L_{eq.}) = (b)(L_{eq.}) =$ Surface area caused due to buoyant force at the water-line of the vessel.

The Eq. (ii) is the wedge-moment (total) by the Direct Method as described. It is a combination of the water characteristic(ρg), the physical dimension(A) & the tilt angle. It is interestingly found the M_w equation, of Eq. (i), is independent of the depth(d) of vessel, although the depth is one of the responsible factors in creating the Meta-center or Meta-centric height of vessel.

Indirect Method:

This method applies the certain mathematical approach on to the Eq. (i)

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aiming at the inclusion of the particular dimensional parameter – the moment of inertia(I). The term *'indirect'* is applied here to involve the concepts of how the I-value evolves out & appears to make its effects on the GM.

$$L\left[6(x^2dx)\left(\frac{1}{dx}\right)+6(xdx)+2(dx)(dx)\right]$$

$$=\oint\left[3(2x^2dA)\left(\frac{1}{dx}\right)+3(2x^2dA)\left(\frac{dx}{x}\right)\left(\frac{1}{dx}\right)+(2x^2dA)(x^{-2}dx)\right]$$

$$=\left[\left(\frac{3}{b}\right)I+\left(\frac{3}{2b}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}-\left(\frac{2}{b}\right)(I)\right]$$

Eq. (i), the total wedge moment is found as,

$$(M_w)=\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I+\left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\theta)\cos\theta$$

... Eq. (iii)

where, the Elemental Area at the Free-water surface $= dA = (L_{eq.})\,dx$; Total Area $= \int (L_{eq.})dx = \int dA = A$

Eq. (iii) is the derived expression of (M_w) finally by the Indirect method &, it's significantly & characteristically defining feature of the Indirect method. It shows the physical parameters more pragmatically involved in the phenomenon while the vessel's tilted. This method incorporates the log-variation which combines with the I-value into the defining final equation of (M_w) which gives idea of effects of the combination on to it & the physical association subsequently during the operation of tilt.

It is therefore clearly seen that there'll be various basic general forms in determining the equation of GM which are given in the Table 1. **(Ref Table – 1)**

Now, besides the wedge-moment., the other side in forming the GM height is the displacement of the position of center of buoyancy 'P' of vessel which is described as follows –

Evaluation of M_m:

In most of the vessel, due to the rotation or displacement the shifting of the centre of buoyancy (P) occurs. The original line of G & P changes to some new (control) line by the displacement angle(θ). The center of gravity (G) does not although change to anywhere else. The point M, caused by the rotation, about which the entire vessel starts to rotate till rest, depending on the magnitude of the rotation is termed as the Meta-center (M). Thereby, formation of meta-center is a functionary element of the

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

displacement or rotation(θ) given. For the formation of this meta-center, a triangle (ΔMP_1P) is formed (Fig.1b & Fig.2) where P_1 is the shifted point of its original position/point P whose varying formation by position with respect to the G-P line is the governing criteria in determining the GM height of vessel in this study. Afterwards with the receding tendency of vessel to become stable or to rest, the rate of change of shifting (from P to P_1or reverse) is although to be fixed & particular for a fixed & definite angle of rotation(θ). On this basis & by the nature the triangle(ΔMP_1G) so formed, there different kinds of the position of P found & described in this study which are, in particular, responsible for the creation of the pursuitous meta-centric height [Ref.6].

In this study, in theΔMP_1P, the angle MPP_1 & the angle MP_1P have been considered to be the angle of criterion of GM height (in terms of PM height) in connection with the wedge-moment. That is why for giving this individual implication in their own identity of the angles PM's finding have been done with the suitable combination of tilt, in the form sine or tangent, & the position of P by notation like Φ or ψ or 90°or else, etc. as applicable. The symbols representing for the angle MP_1P are uniquely individual by nature, feature & identity & indeed of case-sensitive use. **(Ref Figure – 2)**

From ΔMP_1P of Fig.1b & Fig.2 by arc concept, $PP_1 = PM(\theta)$; $(\theta) = \frac{PP_1}{PM}$; PM = Distance between the P & M. The moment evolved out due to the formation of the meta-centre $M_m = (F_b)(PP_1) = (W_b)(PP_1)$; where,$F_b =$ Force of buoyancy; Weight of vessel $= W_b =$ (ρg)Volume of the vessel submerged in water= (ρg)(¥); ¥ = Volume of the submerged portion of vessel & $PP_1 =$ shifted length by the buoyant force F_b. The linear distance PP_1 is the shifted distance due to the formation of meta-center (M) by the rotation(θ). It's to be noted here that the mathematical formulation has been made on the linear basis in the value of PP_1 itself.

So far as this study's mathematical approach is known, a vessel after having been applied by rotation, also of varying magnitudes, is found to have in subjection of eight cases of possibility to form the meta-center (M). These eight cases encompass the possible ways of forming the meta-center for the vessel (Fig.1 & Fig.2), except otherwise stated for the strictness of physical concern. For information, the shift (PP_1) as shown in

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

figures of this study here is by straight-line variation. The Eight (8) Cases are –

Case A. When $\theta = \tan\theta$ & angle $MPP_1 = 90$ deg. **Case C.** When $\theta = \tan\theta$ & angle $MPP_1 = \Phi$ deg.

Case B. When $\theta = \sin\theta$ & angle $MPP_1 = 90$ deg. **Case D.** When $\theta = \sin\theta$ & angle $MPP_1 = \Phi$ deg.

Case E. When $\theta = \tan\theta$ & angle $MP_1P = \psi$ deg. **Case G.** When $\theta = \tan\theta$ & angle $MP_1P = 90$ deg.

Case F. When $\theta = \sin\theta$ & angle $MP_1P = \psi$ deg. **Case H.** When $\theta = \sin\theta$ & angle $MP_1P = 90$ deg.

The cases, Case A, Case B, Case C & Case D are specifically related to the Fig.1b only & the Fig.2 relates to the Case E, Case F, Case G & Case H. The position of the point P & P_1 with respect to each of these, in either position of Fig.1b & Fig.2, is once again to be said as the one of the formulative grounds in deriving the GM height (Table 2) through the so-formed 8 cases of this study – the other factor is the angle FNQ as explained earlier.

Thereby the general form of meta-centric moment, $M_m = (W_b)(PP_1) = (\rho g)(¥)(PP_1)$ ... Eq. (iv)

Table 2 gives the case-wise equation of the meta-centric moment of vessel (Fig.1 & Fig.2).

Respectively, the Eq.(ii) & Eq.(iii) as obtained & regarded as the corresponding formulas of the direct & indirect method shall be now subsequently applied for each case (of the 8 cases, given above) owing to the regards of stable floatation of vessel, indeed, under tilts & it be equated with the Table 2 of the meta-centric moment, to determine the ‘desired’ Meta-centric height simultaneously.

Methodological determination of equations of the Cases (i.e. the 8 Cases) for PM, found & described afterwards, should be primarily of theoretical interests & its applications are solely subject to the viability so far its physicality’s concerned of. The detail explanation is thereby proceeded with by the following. While each of the cases to be in discussion separately, the corresponding valid requirements must always be of the Table 1 & Table 2. **(Ref Table – 2)**

Case-wise Estimation of PM distance:

All the cases are specific to the situations as stated as given in the Table 2 & it’s to be applied on the general equations as derived in the Eq. (ii) & Eq. (iii) subjectively. Cases are hereby derived in the following discussion. Based on the derivations as shown earlier, the derived equations of the PM for each of the cases

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

are the outcomes & the objective of this study. While each case to be derived & be thought of, the pertinent Tables, Figures need to be consulted in, on necessity.

Case A: When $\theta \cong \tan\theta$ & the angle $MPP_1 = 90$ degree

In the Case A, θ is approximately equaled to tanθ & the angle MPP_1 formed is to be equal to 90 degree. This is the conditional case for the determination of GM for case A. The general equation as obtained earlier by the methods, Direct & Indirect, is going to be now applied with the case-specific condition for Case A. In order to determine the PM distance, these are to be equated with the case-implied form of the meta-centric moment given by the Table 1 & Table 2 for Case A. The equation of this simultaneous transformation gives the case-wise distance of GM by the angular aspect.

Derivation given for Case A: In each method the (M_w) is equated with (M_m) to find out PM & GM. Implication of the angular dimension (θ) is to be always by equating each of the Eq. (ii) & Eq. (iii) with the general condition of (M_m) of Table 2.

By Direct method –

By equating the case-implied form of the Eq. (ii) & Eq. (iv),

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\tan\theta)\cos\theta = (\rho g)(¥)(PM)(\tan\theta)$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)(\cos\theta)$

By Indirect method –

By equating the case-implied form of the Eq. (iii) & Eq. (iv),

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\tan\theta)\cos\theta = (\rho g)(¥)(PM)(\tan\theta)$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)(\cos\theta)$ ... Eq. (vi)

In this way, the case-specific nature of the equation of PM distance needs to be determined for the other cases such as Case B, C, D, E, F, G & H & these findings are determined as follows:

Case B: When $\theta = \sin\theta$ & the angle $MPP_1 = 90$ deg.

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

Similarly as determined in the Case A, the PM expression is to be derived here in this Case B by the following way of the Direct & Indirect Method –

By Direct method –

By equating the Eq. (ii) with the Eq. (iv) by implementing the condition of the Case B,

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\sin\theta)(\cos\theta) = (\rho g)(¥)(PM)(\tan\theta)$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)(\cos^2\theta)$

By Indirect method –

By equating the Eq. (iii) with the Eq. (iv) by implementing the condition of the Case B,

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\sin\theta)(\cos\theta) = (\rho g)(¥)(PM)(\tan\theta)$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)(\cos^2\theta)$ ... Eq. (viii)

Case C: When θ = tanθ & angle MPP_1 = Φ deg.

By Direct method –

By equating the Eq. (ii) with the Eq. (iv) by implementing the condition of the Case C,

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\tan\theta)(\cos\theta) = (\rho g)(¥)\left\{\frac{\sin\theta}{\sin(\theta+\Phi)}\right\}PM$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)\sin(\theta + \Phi)$ ... Eq. (vii)

By Indirect method –

By equating the Eq. (iii) with the Eq. (iv) by implementing the condition of the Case C,

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\tan\theta)(\cos\theta) = (\rho g)(¥)\left\{\frac{\sin\theta}{\sin(\theta+\Phi)}\right\}PM$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)\sin(\theta + \Phi)$ ... Eq. (x)

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

Case D: When $\theta = \sin\theta$ & angle $MPP_1 = \Phi$ deg.

By Direct method –

By equating the Eq. (ii) with the Eq. (iv) by implementing the condition of the Case D,

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\sin\theta)(\cos\theta) = (\rho g)(¥)\left\{\frac{\sin\theta}{\sin(\theta+\Phi)}\right\}PM$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)\sin(\theta + \Phi)(\cos\theta)$

By Indirect method –

By equating the Eq. (iii) with the Eq. (iv) by implementing the condition of the Case D,

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\sin\theta)(\cos\theta) = (\rho g)(¥)\left\{\frac{\sin\theta}{\sin(\theta+\Phi)}\right\}PM$$

As $(\rho g) \neq 0$,

- $PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)\sin(\theta + \Phi)$ ... Eq. (xii)

Case E: When $\theta = \tan\theta$ & angle $MP_1P = \psi$ deg.

By Direct method –

By equating the Eq. (ii) with the Eq. (iv) by implementing the condition of the Case E,

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\tan\theta)(\cos\theta) = (\rho g)(¥)\left(\frac{\sin\theta}{\sin\psi}\right)PM$$

... Eq. (xi)

As $(\rho g) \neq 0$,

- $PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)(\sin\psi)$

By Indirect method –

By equating the Eq. (iii) with the Eq. (iv) by implementing the condition of the Case E,

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\tan\theta)(\cos\theta) = (\rho g)(¥)\left(\frac{\sin\theta}{\sin\psi}\right)PM$$

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

As $(\rho g) \neq 0$,

➢ $$PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)(\sin\psi) \quad \text{... Eq. (xiv)}$$

<u>Case F:</u> When $\theta = \sin\theta$ & angle $MP_1P = \psi$ deg.

By Direct method –

By equating the Eq. (ii) with the Eq. (iv) by implementing the condition of the Case F,

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\sin\theta)(\cos\theta) = (\rho g)(¥)\left(\frac{\sin\theta}{\sin\psi}\right)PM$$

As $(\rho g) \neq 0$,

➢ $$PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)(\sin\psi)(\cos\theta)$$

By Indirect method –

By equating the Eq. (iii) with the Eq. (iv) by implementing the condition of the Case F,

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\sin\theta)(\cos\theta) = (\rho g)(¥)\left(\frac{\sin\theta}{\sin\psi}\right)PM$$

As $(\rho g) \neq 0$,

➢ $$PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)(\sin\psi)(\cos\theta) \quad \text{... Eq. (xvi)}$$

<u>Case G:</u> When $\theta = \tan\theta$ & angle $MP_1P = 90$ deg.

By Direct method –

By equating the Eq. (ii) with the Eq. (iv) by implementing the condition of the Case G,

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\tan\theta)(\cos\theta) = (\rho g)(¥)PM(\sin\theta)$$

As $(\rho g) \neq 0$,

➢ $$PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right) \quad \text{... Eq. (xv)}$$

By Indirect method –

By equating the Eq. (iii) with the Eq. (iv) by implementing the condition of the Case G,

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\tan\theta)(\cos\theta) = (\rho g)(¥)PM(\sin\theta)$$

As $(\rho g) \neq 0$,

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

- $$PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right) \quad \ldots \text{Eq. (xviii)}$$

Case H: When $\theta = \sin\theta$ & angle $MP_1P = 90$ deg.

By Direct method –

By equating the Eq. (ii) with the Eq. (iv) by implementing the condition of the Case H,

$$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\sin\theta)(\cos\theta) = (\rho g)(¥)PM(\sin\theta)$$

As $(\rho g) \neq 0$,

- $$PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)(\cos\theta) \quad \ldots \text{Eq. (xix)}$$

By Indirect method –

By equating the Eq. (iii) with the Eq. (iv) by implementing the condition of the Case H,

$$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I + \left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\sin\theta)(\cos\theta) = (\rho g)(¥)PM(\sin\theta)$$

As $(\rho g) \neq 0$,

- $$PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)(\cos\theta) \quad \ldots \text{Eq. (xx)}$$

Now, from the Fig.1 & Fig.2, GM = (PM – PG) & using it, the Meta-centric height (GM) of vessel by the angular aspect can be determined. The value or expression of the (PG) is to be of fixed quantity for a vessel. For each case of the Case A to Case H, the corresponding value of PM as well as GM can be estimated from the derivations so obtained by cases. The Table 3 shows the detailing of the PM values so obtained for the 8 cases. Both the methods (Direct & Indirect) deliver the equation of GM which although need 'useful' justification subjectively through required mathematical synthesis, model analysis, & etc. further, for both of its inter-related validity & their individual acquaintanceship for physical viability & more. In this way, the GM expression for a vessel is determined by this study where a more precise way has been taken & implemented to derive the several conditional GM equations resulting into better precision in its derived equations, values for all possible situations of a vessel. Based on these findings as given in the Table 3, it is now easily possible to view the vision of what the condition & its mathematical formation would be.

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

5. DIMENSIONAL STUDY

In this section the finding of this study has been discussed further by dimensional analysis in which the dimensional relation in between various variables associated with the finding has been determined. This means, the detail insights inside of the equation found in this study are analyzed dimensionally [3] in order to have the meaning of their inter-relationship so existing in. Following is the detail explanation given along with its necessary formulation & finding –

Here the finding as determined by the Direct & Indirect method for all the cases as discussed is required to be analyzed, individually for each of the Cases. For the sake of understanding & limited space, only one equation, Eq. (vi), is here explained with the necessary study –

DIMENSIONAL STUDY for the Case A:

The functional expression of the Meta-centric height (GM) from the Eq. (vi) for the Case A (By the Direct Method) is,

$$\mathrm{PM} = \left(\frac{4}{3}\right)\left(\frac{100}{\mathrm{P_p}}\right)\left(\frac{\mathrm{Ab}}{¥}\right)\cos\theta$$

Assuming, $\mathrm{E} = \left(\frac{4}{3}\right)\left(\frac{100}{\mathrm{P_p}}\right)$ & $(1+\mathrm{F}^2) = \left(\frac{\mathrm{Ab}}{¥}\right)$; where, F is called here as Similitude Factor of vessel.

Thereby, $\mathrm{PM} = \mathrm{E}(1+\mathrm{F}^2)(\cos\theta)$

Now, as per the law of geometrical similitude of the dimensional study, the following scale ratios are obtained:-

In doing the dimensional justification, model & its prototype of a vessel as shown in the Fig.1a are herein assumed & designated by the subscript of the letter 'm' & 'p' respectively. And, the scale ratio (i.e., prototype to model) is termed by the subscript of the letter 'r', correspondingly.

Thereby, Scale ratio for length (i.e., simply, Scale ratio) = $L_r = L_p/L_m$

Scale ratio for area (i.e. Area ratio) = $A_r = A_p/A_m$

and, Scale ratio for volume (i.e. Volume ratio) = $¥_r = ¥_p/¥_m$

Moreover, Scale ratio for the Geometric Similitude Factor = Fr $= \frac{\mathrm{Fp}}{\mathrm{Fm}}$

Now, for the expression to be said with respect to the dimensional similitude as obtained, the scale ratio for the geometric meta-centric height is given by,

$$(\mathrm{PM})\mathrm{r} = \left[\frac{(\mathrm{PM})\mathrm{p}}{(\mathrm{PM})\mathrm{m}}\right]$$

$$= (\mathrm{Er})\left[\frac{1+\mathrm{F_p^2}}{1+\mathrm{F_m^2}}\right](\cos\theta\mathrm{r})$$

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

$$= (Er)\left[\frac{1 + F_m^2 + F_p^2 - F_m^2}{1 + F_m^2}\right](\cos\theta r)$$

$$= (Er)\left[1 + \frac{F_p^2 - F_m^2}{1 + F_m^2}\right](\cos\theta r)$$

(PM)r

$$= (Er)\left[1 + \frac{F_r^2 - 1}{\left(\frac{1}{F_m^2}\right) + 1}\right](\cos\theta r) \qquad \ldots (vi.a)$$

where, $Fr = \left(\frac{Fp}{Fm}\right)$; $Er = \left(\frac{Ep}{Em}\right)$ &, $\theta r = \left(\frac{\theta p}{\theta m}\right)$ = Dimensional Scale Ratios.

The expression Eq. (vi.a) is self-explanatory & is the required expression for its further analysis for preparing its corresponding modeling structures of the desired prototypes (or reverse), in order to get the desired results as the case may be for different configurations of vessel.

Now, an exemplary estimation is given for vessel rectangular in plan with 'b' as width & 'd' as the depth of submergence of the vessel (Fig.1a). The factor $\left(\frac{Ab}{¥}\right)$ is then equal to $\left(\frac{Ab}{¥}\right) = \left(\frac{L_{eq.}b^2}{L_{eq.}bd}\right) = \left(\frac{b}{d}\right)$

For this rectangular vessel, the similitude factor is to be obtained as

$$(1 + F^2) = \left(\frac{Ab}{¥}\right) = \left(\frac{b}{d}\right)$$

$$F^2 = \left(\frac{b}{d}\right) - 1 = \left(\frac{b - d}{d}\right)$$

The value of F^2 is 1, when b = 2d.

The value of F^2 is more than 1, when b >2d.

The value of F^2 is 0 when b = d.

The value of F^2 is < 1 when b < d.

With this evaluative determination in the significance of the factor F, the Eq.(vi.a) may be used suitably so far as the dimensional configuration if vessel is concerned. The stability of vessel should lie in the range of the values of $(1+F^2)$ so found. **(Ref Table – 3)**

Thereby, the way of formulation as given in the Eq. (viii) as derived for the scale ratios for dimensional analysis may be kept similar for all the cases (Case B, C, D, E, F, G, H; Table 3) discussed in this paper or it may be varied, suitably, in the use.

6. RESULTS & DISCUSSION

1) The PM equations as obtained from the Eq.(v) to the Eq.(xx), may be represented graphically to have the profile or behavior in the pattern of the PM value by the angle of

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

oscillation, i.e., the tilt(Θ) and/or the other angles(such as Φ, ψ) so considered. Here an exemplary representation is given for the Eq.(vii) of the Case B in its Direct Method,

The equation of PM, as per the Eq.(vii), is as,

$$\mathrm{PM} = \left(\frac{4}{3}\right)\left(\frac{100}{\mathrm{P_p}}\right)\left(\frac{\mathrm{Ab}}{¥}\right)(\cos^2\theta)$$

Now this equation for graphical plotting is written as

$$\mathrm{PM} = \mathrm{D_f}\,(\cos^2\theta)$$

where, Dimensional factor of vessel = $\mathrm{D_f} = \left(\frac{4}{3}\right)\left(\frac{100}{\mathrm{P_p}}\right)\left(\frac{\mathrm{Ab}}{¥}\right)$.

Considering the D_f as constant for a vessel, a graph can be plotted with θ values along the abscissa & the PM values along the ordinate. Table 4 shows a tabulation of values of the tilt (θ) & its corresponding functional values of the PM. A graph is thereby determined & given the Figure 3 to show & understand the behavioral pattern in the value of PM for the equation of the Case B. In this way, various case-wise graphs can be plotted to have this knowledge graphically to decide on the precision level of the equations so obtained of the PM.

2) Inclusion of the areal section of the wedge-strips by the angular measure has been considered here as a way in finding the M_w, by avoiding the assumption (v), keeping the other necessary determination unaltered. **(Ref Table – 4) & (Ref Figure – 3)**

3) Usefulness of the GM height is utmost important. Properly designed vessel requires the various variables to be well verified & justified in order to have it practically feasible. Suitable case or cases, out of the 8s (Table 3), should be the particular basis. Vessels need to be case-wise. The dimension like L, b, d, I value etc. requires subsequent estimation individually.

4) The case-sensitive conditions/equations of vessel for the different conditions of vessel as shown in the Fig.1 & Fig.2 does also give a way of providing the derivation of the GM height, in addition to the earlier point (1) or separately also.

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

7. CONCLUSION (with Future Scope)

Following conclusions are hereby given along with its possible future scope of work –

1) The case-wise estimation of GM height is entirely matter of theoretical interests. Some may be discarded & some must be of useful kind.

2) As most of the derived equation of PM is a function of the factor(I/¥), the PM as well as the GM heights may now be viewed by comparison in between or amongst by all the cases with the conventional equation of it, as explained earlier, in order to have the correct knowledge and/or the case or the cases be so involved in, for a particular vessel of desired one also.

3) The way the equation of GM has been determined shall have to be useful in deriving the energy as given in its future aspect of determining the work done by the concern of angular aspect.

Future scope: Depending on the oscillating behavior of vessel, the work-done of vessel should represent the amount of energy produced. Keeping the (case-wise) methodological description of this study, following is the future scope of it which is nonetheless a very common application of mechanics –

Let, n = no. of revolution of vessel; for a complete oscillation, total = 2nπ

M = Moment given or the moment upon which vessel oscillates; &, t = time required (on one complete revolution of the vessel). Maximum work done (on one-circle complete revolution) = $\left\{\frac{2\pi(n)M}{t}\right\}$ watt; subject to the variability in values of the moment subjectively. This equation of work done gives a particular outcome which is called the energy, produced during the oscillation of vessel. Regarding the value of M, the particular situation of the vessel & its corresponding GM height (Table 3) shall reckon on the magnitude of the energy produced & the intensity of the energy such produced depends on the condition of the vessel as shown in the Fig.1 & Fig.2. The value of M in the energy equation (i.e., work done) is to be determined using the Eq.(iv)

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

after finding out & selecting the PM value of vessel under a particular situation.

With all these, this study has thereby completed its goals, achieved the all the possibility of (stable) floating & brought the subject of the GM height to a different world of perspectives where the water-transport facility should be into a world soon, full of variety of vessels floating along troubled path ever, with ease of control by making incorporated of this study's outcome from small scale to large scale to voyaging vehicles like the vessel [Ref. 7, 8 & 9].

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ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

LIST OF TABLES

Table 1. General Equation of the Total Wedge Moment in the formulation of the PM Distance[a]

Method	**Total Wedge Moment** (M_w)
Direct Method	$\left(\frac{4}{3}\right)\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}(Ab)(\theta)(\cos\theta)$
Indirect Method	$\left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[I+\left(\frac{3}{2}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\theta)(\cos\theta)$

[a]Meta-centric Moment, $M_m = (W_b)(PP_1)$

Table 2. General Condition of the Meta-centric Moment in the Formulation of GM (Case-wise)

Case	**Meta-centric Moment** $(\mathbf{M_m})$		
	Mode of Angles	$\mathbf{PP_1}$	$(\mathbf{M_m})$
Case A	$(\theta) = \tan(\theta); \angle MPP_1 = 90°$	$PM(\theta) = PM(\tan\theta)$	$(\rho g)(¥)(PM)(\tan\theta)$
Case B	$(\theta) = \sin(\theta); \angle MPP_1 = 90°$	Same as Case B	Same as Case B
Case C	$(\theta) = \tan(\theta)$; $\angle MPP_1 = \Phi$	$\left\{\frac{\sin\theta}{\sin(\theta+\Phi)}\right\}PM$	$(\rho g)(¥)\left\{\frac{\sin\theta}{\sin(\theta+\Phi)}\right\}PM$
Case D	$(\theta) = \sin(\theta); \angle MPP_1 = \Phi$	Same as Case C	Same as Case C
Case E	$(\theta) = \tan(\theta)$; $\angle MPP_1 = \psi$	$\left(\frac{\sin\theta}{\sin\psi}\right)PM$	$(\rho g)(¥)\left(\frac{\sin\theta}{\sin\psi}\right)PM$
Case F	$(\theta) = \sin(\theta); \angle MPP_1 = \psi$	Same as Case E	Same as Case E
Case G	$(\theta) = \tan(\theta); \angle MP_1P = 90°$	$PM(\theta) = PM(\sin\theta)$	$(\rho g)(¥)PM(\sin\theta)$

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

Case H	$(\theta) = \sin(\theta); \angle MP_1P = 90°$	Same as Case G	Same as Case G

Table 3. Equation of PM Distance of the Meta-centric Height (GM)

Case	Direct Method	Indirect Method
Case A	$PM = \left(\frac{4}{3}\right)\left(\frac{100}{P_p}\right)\left(\frac{Ab}{¥}\right)(\cos\theta)$	$PM = \left(\frac{100}{P_p}\right)\left\{\left(\frac{1}{3b}\right)(L_{eq.})\right\}\left[1 + \left(\frac{3}{2I}\right)\log_e\left(\frac{b}{2}\right)\right]\left(\frac{I}{¥}\right)(\cos\theta)$
Case B	$PM = (\cos\theta) *$ Case A	$PM = (\cos\theta) *$ Case A
Case C	$PM = \left(\frac{4}{3}\right)\left\{\left(\frac{100}{P_p}\right)\right\}\left(\frac{Ab}{¥}\right)\sin(\theta + \Phi)$	$PM = \left\{\left(\frac{100}{P_p}\right)\right\}\left[\left(\frac{I}{¥}\right) + \left(\frac{3}{2¥}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})\sin(\theta + \Phi)$
Case D	$PM = (\cos\theta) *$ Case C	$PM = (\cos\theta) *$ Case C
Case E	$PM = \left(\frac{4}{3}\right)\left\{\left(\frac{100}{P_p}\right)\right\}\left(\frac{Ab}{¥}\right)(\sin\psi)$	$PM = \left\{\left(\frac{100}{P_p}\right)\right\}\left[\left(\frac{I}{¥}\right) + \left(\frac{3}{2¥}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})(\sin\psi)$
Case F	$(\cos\theta) *$ Case E	$(\cos\theta) *$ Case E

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

Case G	$PM = \left(\frac{4}{3}\right)\left\{\left(\frac{100}{P_p}\right)\right\}\left(\frac{Ab}{¥}\right)$	$PM = \left\{(\rho g)\left(\frac{100}{P_p}\right)\right\}\left[\left(\frac{I}{¥}\right) + \left(\frac{3}{2¥}\right)\left\{\log_e\left(\frac{b}{2}\right)\right\}\right]\left(\frac{1}{3b}\right)(L_{eq.})$
Case H	$(\cos\theta) * \text{ Case G}$	$(\cos\theta) * \text{ Case G}$

Table 4. Value of PM versus Angle of Tilt (θ)

Sl.	θ	PM
0	0°	1
1	3°	0.997
2	6°	0.989
3	9°	0.975
4	12°	0.956
5	15°	0.933
6	18°	0.904
7	21°	0.87
8	24°	0.834
9	27°	0.793
10	30°	0.75
11	33°	0.703
12	36°	0.654
13	39°	0.604
14	42°	0.552
15	45°	0.5
16	48°	0.447
17	51°	0.39
18	54°	0.345
19	57°	0.296
20	60°	0.25
21	63°	0.206
22	66°	0.165
23	69°	0.128
24	72°	0.095
25	75°	0.067
26	78°	0.043
27	81°	0.024
28	84°	0.011
29	87°	#

#*indicates that it gets lowered by 0.05 units in each subsequent interval.*

LIST OF FIGURES

Figure - 1 – Meta-centric height of Vessel

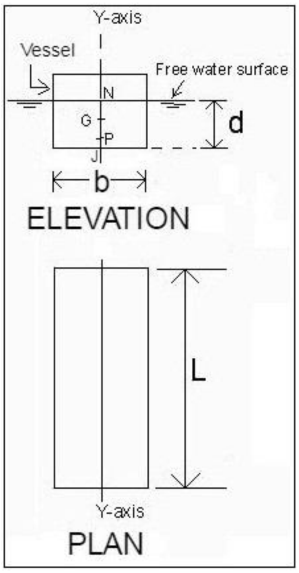

Fig.1a – Plan & Section View of Vessel.

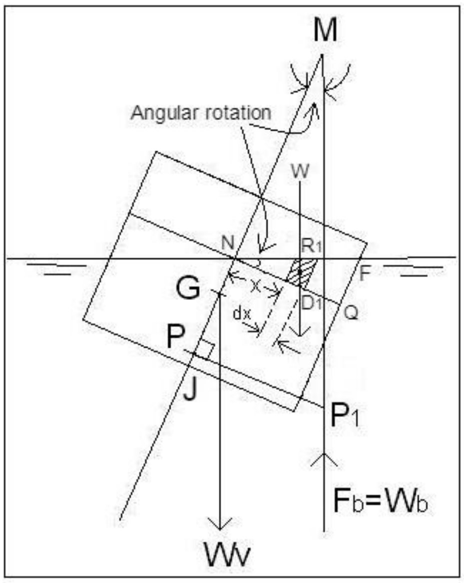

Fig.1b – Vessel, when tilted (Case A, B, C & D).

ISSN 2394-9341 (Online); Volume 5 Issue 1 (2019)

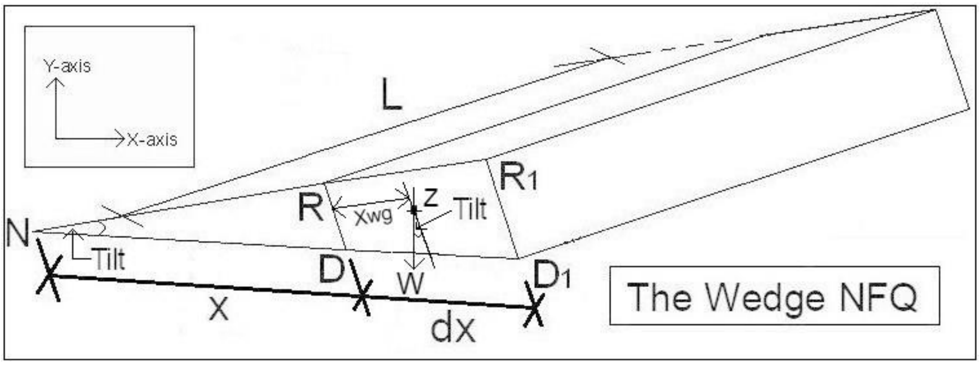

Fig.1c – Dimensional View of the segmental wedge (in Tilted Vessel).

Figure- 2 – Meta-centric height of vessel (Case E, Case F, Case G & Case H)

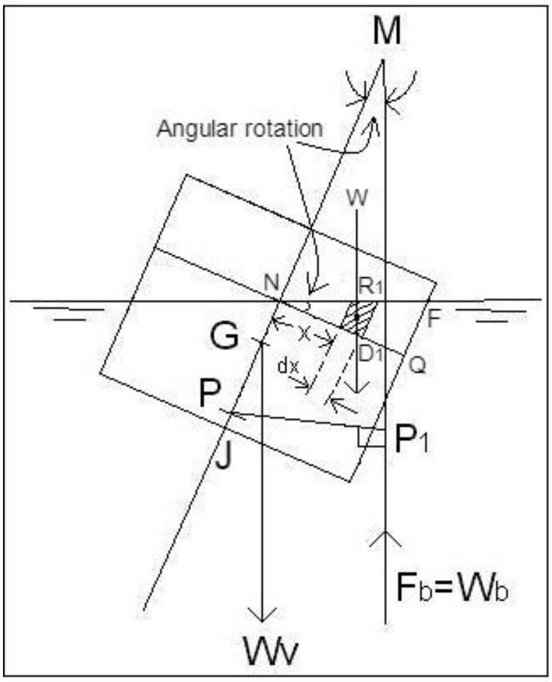

Figure 3: Profile of PM Height