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Free Vibration of Metallic Beam under Various Boundary Conditionsusing mathematical approach

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Abstract*:* This paper represents analytical and numerical investigation of Dynamic characterization for squared and rectangular metallic beams for four different boundary conditions like, Clamped-Clamped(C-C), Clamped-Simply Supported(C-SS), Clamped-Free(C-F) and Simply Supported-Simply Supported(SS-SS). Analytical solution is carried out using Euler-Bernoulli beam theory and Newton Raphson Method. Initially, the equations of motion are provided and solutions including the effects of the geometric boundary conditions and natural boundary conditions are obtained for the first three modesof natural frequency. To confirm the reliability of the analytical vibration analysis carried out in the present work, the finite element method (FEM) is employed to discretize the model and obtain a numerical approximation of the motion equation.Numerical results are presented in tabular form to figure out the effects of boundary conditions on the dynamic characteristics of the beam. The above mention effects play very important role on the dynamic behavior of the beam and found a good agreement with them.

Keywords: Beam, natural frequency, boundary condition, FEM, Analytical approach.

I. INTRODUCTION

A beam is a slender horizontal structural member that resists lateral loads by bending, and this important element of engineering structures appears in various forms and comprises various artifacts, such as supporting members in high-rise buildings, railways, long-span bridges, flexible satellites, gun barrels, robot arms, airplane wings, etc. [1]In many engineering applications, beams are subjected to dynamic loads, which can excite beam structural vibrations and cause durability concerns or discomfort because of the resulting noise and vibration. In addition, if the vibration exceeds certain limits, there is the danger of beam breakage or failure due to resonance. Due to beams are important structural elements, vibration analysis has been a vital task in their design for engineers and researchers for more than a century [3, 4, 5]. Then an increasing interest has been Observed regarding the vibration of beams, and several studies have appeared in the general literature [6].

In this study, the free vibrations of square and rectangular cross-sectioned stainless steel beams are investigated analytically and numerically under four different boundary conditions. Analytical solution is carried out using Euler-Bernoulli beam theory, in which material is assumed to be linear-isotropic, and Newton Raphson Method. This method is based on the simple idea of linear approximation, and used for finding the roots of equations. It is particularly useful for transcendental equations, composed of mixed trigonometric and hyperbolic terms. Such equations occur in vibration analysis. An example is the calculation of natural frequencies of continuous structures [2]. Solutions including the effects of the geometric characteristics, i.e., length and cross sectional area, and boundary conditions are obtained and discussed for the natural frequencies of the first three modes. Furthermore, to confirm the reliability of the vibration analysis carried out in the present paper as well, all the analytical results are checked with the corresponding numerical results obtained from Finite Element Method (FEM)-based software called ANSYS [7], where the method is established on the idea of building a complicated object with simple blocks, or, dividing a complicated object into smaller and manageable pieces [8].

The ANSYS provides several mode extraction methods like block Lanczos, subspace, power dynamics, reduced, asymmetric, damped, QR damped. Note that the default mode extraction method chosen is the Reduced Method. This is the fastest method as it reduces the system matrices to only consider the Master Degrees of Freedom. The Subspace Method extracts modes for all DOF's. It is therefore more exact but, it also takes longer to compute (especially when the complex geometries). This Lanczos is the recommended method for the medium to large models. In addition to its reliability and efficiency, the Lanczos method supports sparse matrix methods that significantly increase computational speed and reduce the storage space.

This method also computes precisely the eigenvalues and eigenvectors [9]. Present analysis can be used as a comparative study or data for thedifferent solution methods of future works in the relatedfield.

TABLE 1.Nomenclature

Nomenclature	
f	Natural Frequency
α_n	Frequency parameter
E	Modulus of elasticity
I	Moment of inertia of section
A	Effective cross section area
l	Effective length of beam
ρ	Density of material

II. BASIC EQUATION FOR BEAM VIBRATION

If the cross-sectional dimensions of the beam are small compared to its length, the system is known as Euler-Bernoulli beam. Only the thin beams are treated under it. The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross sectional area A, Flexural rigidity EI and density of material ρ. Element dx of beam is subjected to shear force Q and bending moment M.

Using Euler-Bernoulli beam theory, one can obtain the equation of motion of a beam with homogeneous material properties and constant cross section as follows [10, 11]:

$$EI\frac{\partial^4 y}{\partial x^4}+(k)\frac{\partial^2 y}{\partial t^2}=0 \quad (1)$$

Where , $k = \rho A$ is the linear mass density of the beam?

The solution of the (1) is sought by separation of variables. Assume that the displacement can be separated into two parts: one is depending on the position and the other is depending on time, as follows:

$$w(x,t)=\Lambda(x)\Psi(t) \quad (2)$$

Where Λ and Ψ are independent of time and position respectively.

Substituting (1) into (2) and after some mathematical rearrangements, the following equation is obtained:

$$-\frac{EI}{k\Lambda(x)}\frac{\partial^4\Lambda(x)}{\partial x^4}=\frac{1}{\Psi(t)}\frac{\partial^2\Psi(t)}{\partial t^2} \quad (3)$$

As observed from (3), the left side depends on the variable x, and the right side depends on the variable t, as previously noted. Consequently, the variables have been separated, and each side of (3) must equal a constant, denoted $-\omega^2$to have simple harmonic motion in the system.

$$-\frac{EI}{k\Lambda(x)}\frac{\partial^4\Lambda(x)}{\partial x^4}=\frac{1}{\Psi(t)}\frac{\partial^2\Psi(t)}{\partial t^2}=-\omega^2 \quad (4)$$

If the position variable is separated

$$\frac{\partial^4\Lambda(x)}{\partial x^4}-\delta^4\Lambda(x)=0 \quad (5)$$

Where,

$$\delta^4=\omega^2\frac{k}{EI}, \quad (6)$$

If the time variable is separated,

$$\frac{\partial^2\psi(t)}{\partial t^2}+\omega^2\psi(t)=0 \quad (7)$$

Eq. (5) is solved as follows:

$$\Lambda(x)=C_1\sinh\delta x+C_2\cosh\delta x+C_3\sin\delta x+C_4\cos\delta x \quad (8)$$

Where, C_1, C_2, C_3, C_4 are constants, and sinh and cosh are the hyperbolic sin e and cos e functions, respectively.

Eq. (7) is solved as follows:

$$\Psi(t)=C_5\sin\omega t+C_6\cos\omega t \quad (9)$$

Where, C_5 and C_6 are constants.

Thus, if (8) is multiplied by (9) to obtain$w(x,t)$, it yields eight combined constants as:

$$w(x,t)=\left(C_1\sinh\delta x+C_2\cosh\delta x+C_3\sin\delta x+C_4\cos\delta x\right)*(C_5\sin\omega t+C_6\cos\omega t) \quad (10)$$

Where, the constants C_1, C_2, C_3, C_4 can be obtained from the boundary conditions, and C_5, C_6 can be obtained from the initial conditions

Finally, using (7) the natural frequency the beam is found as follows:

$$f=\frac{1}{2\pi}\alpha_n^2\sqrt{\frac{EI}{A\rho l^4}} \quad (11)$$

III. SOLUTION OF BASIC EQUATION FOR BEAM VIBRATION

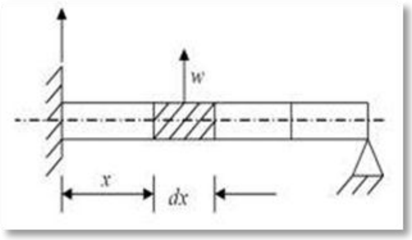

Fig. 1 Horizontal beam geometry

$$w\big|_{x=0}=0 \text{ and } \left.\frac{\partial w}{\partial x}\right|_{x=0}=0, \quad w\big|_{x=L}=0 \text{ and } \left.\frac{\partial w}{\partial x}\right|_{x=L}=0 \tag{12}$$

The non-trivial solution of the determinant of the coefficient matrix is as follows:

$$\cos\delta_n \cosh\delta_n L=1 \tag{13}$$

B.Particular Solution for Clamped-Simply Supported (C-SS) Beam:

The boundary conditions satisfied by a C-SS beam are as follows:

$$w\big|_{x=0}=0 \text{ and } \left.\frac{\partial w}{\partial x}\right|_{x=0}=0, \quad w\big|_{x=L}=0 \text{ and } \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=L}=0 \tag{14}$$

The solution of the determinant of the coefficient matrix is as follows:

$$\tanh\delta_n L=\tan\delta_n L \tag{15}$$

C. *Particular Solution for Clamped-Free (C-F) Beam:*

The boundary conditions satisfied by a C-F beam are as follows:

$$w\big|_{x=0}=0 \text{ and } \left.\frac{\partial w}{\partial x}\right|_{x=0}=0, \quad \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=L}=0 \text{ and } \left.\frac{\partial^3 w}{\partial x^3}\right|_{x=L}=0 \tag{16}$$

A. Particular solution for clamped-clamped (C-C) beam:

The boundary conditions satisfied by a C-C beam are as follows:

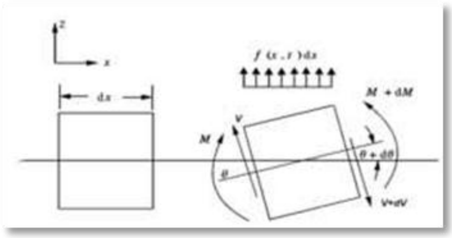

Fig. 2 Cross section of square beam

The non-trivial solution of the determinant of the coefficient matrix is as follows:

$$\cos\delta_n L\cosh\delta_n L=-1 \tag{17}$$

D. Particular Solution for Simply Supported-Simply Supported (SS-SS) Beam:

The boundary conditions satisfied by a SS-SS beam are as follows:

$$w\big|_{x=0}=0 \text{ and } \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0}=0, \quad w\big|_{x=L}=0 \text{ and } \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=L}=0 \tag{18}$$

The non-trivial solution of the determinant of the coefficient matrix is as follows:

$$\sin\delta_n \sinh\delta_n L=0 \tag{19}$$

From the above equations, the first three roots of frequency parameters are calculated using the Newton-Raphson Method, eigenvalues are tabulated in Table 2.

IV. NUMERICAL RESULTS AND DISCUSSION

FEM analysis is also performed for the said constrained condition of square and rectangular beam. This analysis gives the natural frequency for corresponding mode shape. After putting value of frequency in (11), frequency parameter can be found out. Then this value of frequency parameter δL is verified with the value available from mathematical calculations. This gives close match. Results are tabulated in Table 3.

TABLE 2. Value of Frequency Parameter (Δl) Using Newton-Raphson Method

Mode number	C-C	C-SS	C-F	SS-SS
1	4.7300	3.9266	1.8751	3.1415
2	7.5832	7.0686	4.6941	6.2832
3	10.9956	10.210	7.8534	9.4248

TABLE 3.Convergence Studies Of All Possible Boundary Conditions with Different Geometric Characteristics of Beam

For Clamped-Clamped (C-C) Beam

Mode	Rectangular beam dimensions (l*b*h)											
	1750*150*25				2000*150*25				2250*150*25			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error
1	1.33	4.75	4.73	0.41	1.02	4.75	4.73	0.39	0.80	4.75	4.73	0.36
2	3.66	7.88	7.85	0.32	2.80	7.88	7.85	0.32	2.21	7.88	7.85	0.30
3	7.17	11.02	11.00	0.23	5.49	11.02	11.00	0.25	4.34	10.84	11.00	-1.44
Mode	Square beam dimensions (l*b*h)											
	1750*25*25				2000*25*25				2250*25*25			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error
1	1.32	4.73	4.73	0.10	1.01	4.74	4.73	0.11	0.80	4.74	4.73	0.12
2	3.64	7.85	7.85	0.02	2.79	7.86	7.85	0.05	2.20	7.86	7.85	0.07
3	7.12	10.84	11.00	-1.44	5.46	10.99	11.00	-0.04	4.31	11.00	11.00	0.00

For Clamped-Simply Supported (C-SS) beam

Mode	Rectangular beam dimensions (l*b*h)											
	1750*150*25				2000*150*25				2250*150*25			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error
1	0.91	3.94	3.93	0.27	0.70	3.94	3.93	0.25	0.55	3.94	3.93	0.24
2	2.96	7.08	7.07	0.22	2.27	7.08	7.07	0.22	1.79	7.08	7.07	0.21
3	6.17	10.23	10.21	0.15	4.72	10.23	10.21	0.17	3.73	10.23	10.21	0.18
Mode	Square beam dimensions (l*b*h)											
	1750*25*25				2000*25*25				2250*25*25			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error
1	0.92	3.93	3.93	0.10	0.70	3.93	3.93	0.09	0.55	3.94	3.93	0.24
2	2.94	7.07	7.07	0.04	2.26	7.07	7.07	0.06	1.79	7.08	7.07	0.21
3	6.14	10.20	10.21	-0.06	4.71	10.21	10.21	-0.02	3.73	10.23	10.21	0.18

For Clamped-Free (C-F) beam

Mode	Rectangular beam dimensions (l*b*h)											
	1750*150*25				2000*150*25				2250*150*25			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error
1	0.21	1.88	1.88	0.30	0.16	1.88	1.88	0.28	0.13	1.88	1.88	0.26
2	1.31	4.71	4.69	0.25	1.00	4.71	4.69	0.24	0.79	4.71	4.69	0.23
3	3.65	7.87	7.85	0.21	2.80	7.87	7.85	0.22	2.21	7.87	7.85	0.22
Mode	Square beam dimensions (l*b*h)											
	1750*25*25				2000*25*25				2250*25*25			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error
1	0.21	1.88	1.88	0.13	0.16	1.89	1.88	0.67	0.13	1.88	1.88	0.13
2	1.30	4.70	4.69	0.09	1.00	4.70	4.69	0.10	0.79	4.70	4.69	0.10
3	3.64	7.86	7.85	0.03	2.79	7.86	7.85	0.06	2.20	7.86	7.85	0.08

For Simply Supported-Simply Supported (SS-SS) Beam

Mode	Rectangular beam dimensions (l*b*h)											
	1750*150*25				2000*150*25				2250*150*25			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error

1	0.58	3.15	3.14	0.11	0.45	3.15	3.14	0.11	0.35	3.15	3.14	0.11
2	2.33	6.29	6.28	0.09	1.79	6.29	6.28	0.10	1.41	6.29	6.28	0.10
3	5.25	9.43	9.42	0.06	4.02	9.43	9.42	0.08	3.18	9.43	9.42	0.09
Mode	**Square beam dimensions (l*b*h)**											
	1750*25*25				**2000*25*25**				**2250*25*25**			
	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error	f_{FEM}	∂L_{FEM}	∂L_{analy}	% error
1	0.58	3.14	3.14	0.10	0.45	3.14	3.14	0.10	0.35	3.14	3.14	0.11
2	2.33	6.29	6.28	0.04	1.79	6.29	6.28	0.06	1.41	6.29	6.28	0.07
3	5.24	9.42	9.42	-0.04	4.01	9.42	9.42	0.00	3.17	9.43	9.42	0.02

V. CONCLUSION

In this study, the dynamic characteristics of square and rectangular cross-sectioned stainless steel beams are investigated analytically and numerically under four different boundary conditions. Analytical solution is carried out using Euler-Bernoulli beam theory, in which material is assumed Solutions including the effects of the geometric characteristics, i.e., length and cross sectional area, and boundary conditions are obtained and discussed for the natural frequencies of the first three modes only.

- The beam has the highest natural frequencies under clamped-clamped(C-C) boundary conditions.
- The beam has the lowest natural frequencies under clamped-free(C-F) boundary conditions.
- The natural frequencies of the beam decrease with increasing length.
- The natural frequencies of the beam increase with increasing cross sectional area.
- The natural frequencies increase with the increase in mode number.
- Validate the ANSYS results with the mathematical modeling and accuracy of ANSYS results depends upon meshing size, Element type and input information.
- With all possible geometric conditions and boundary conditions, error varies from -1.44% to 0.67%

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