



## CHARACTERIZATION OF WEAR PARTICLES BY DIFFERENT FRACTAL METHODS : A REVIEW

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**Abstract**—Fractals can be very useful when applied to tribology. Fractal descriptions of wear particles require surface texture information to be processed. The morphology of wear particles depends on its surface characteristics. These characteristics are represented by fractal dimension derived from the boundary profile. In this paper, Fractal dimension is used to describe wear particles obtained from engine oil. This article compares different fractal methods to calculate fractal dimension for characterization of wear particles. This paper presents a review of progress and developments in fractal dimension computing methods as applied to characteristics the surface of wear particles. Some of Fractal dimension methods are structure walk method, box counting method etc.

**Keywords**—morphology; texture; boundary profile; fractal dimension

### I. INTRODUCTION

P. Podsiadlo, M. Kuster and G.W. Stachowiak [1] states that wear processes occurring between two interacting surfaces result in the production of wear particles. The structure of the particles, surface textures and boundary shapes are related to wear processes involved in their formation. Ferrography technique for wear particle analysis [2] is useful, but when compared with the various methods of analysis investigated, quantitative morphological analysis offers the most promising prospects. It serves as the basis for accurate characterization of wear particles. Image analysis techniques are available, for example, Fourier analysis, basic shape factors, fractal outline, textural dimensions etc. The size and morphology [3] of wear particles were used in early investigations as the primeval considerations in finding quantitative characterization parameters. Morphological analysis of wear particles is an effective and versatile means of wear debris analysis for machine condition monitoring and fault diagnosis[4]. Since, in general, the boundaries and surfaces of wear particles are

not Euclidean instead exhibiting fractal nature. The development of numerical descriptors, application of computer technology and image analysis methodologies aids the assessment of particle morphology. Numerical descriptors would simplified manifold and would not require specialized expertise. Fractal methods also useful in characterizing the surface anisotropy and directionality[5]. This method also facilitates the description of pigments, powders and wears particles.

The most effective numerical descriptors of morphology appear to be the fractal and Fourier parameters [6]. Fractal parameters represent a profile's texture, applied to fine particles. Fourier descriptors are successfully derived from particles in order to describe their size, shape and texture. Fractal parameters offer simplicity and easy interpretation unlike the Fourier descriptors. Fractal descriptors are simply applied to a section of the boundary. Fractal methods give unique parameters which are independent of resolution. Fractal descriptors have also been applied to characterize fractured surfaces when studied under the scanning electron microscope (SEM) by relating secondary electron current to accelerating voltage. Fractal methods have also been used recently to characterize and model engineering surface profiles.

The aim of this paper is to investigate fractal methods and their applications in different fields. It has been seen that up to now problems related to fractal analysis has not been reviewed in terms of engineering applications. Therefore, here in this present work the basic fundamentals of fractal and their development is discussed. This study will help in judging the best applicable fractal method for a particular application.



1. FRACTAL

Fractal dimensions were introduced by Mandelbrot in 1967, mostly in 1977, According to Mandelbrot [7] the term ‘fractal’ comes from the Latin adjective ‘fractus’, which has the same root as ‘fraction’ and ‘fragment’ and means ‘irregular and fragmented’. Mandelbrot (1982) conceived and developed a new geometry of nature (fractals) and implemented it in diverse fields. It describes many of the irregular and fragmented patterns around us. It provides the possibility of describing and simulating landscapes precisely by using a mathematical model. These strategies are based upon the four methods outlined in Mandelbrot’s book, *Fractals: Form, Chance and Dimension*, for evaluating geographical curves, such as coastlines (Fig 1). These have been described by Peleg et al. and are summarized below [8].

$$(1)$$

Where  $M$  a positive is constant and  $D$  is a constant (at least equal to unity). Richardson observed that  $D$  is a “characteristic of a frontier, (which) may be expected to have some positive correlation with one’s immediate visual perception of the frontier”. (Mandelbrot) named such dimensions as being ‘fractal’.

3. MANDELBROT’S METHOD B

The shortest path on land that is not further than  $e$  from the water can be regarded as an approximate length  $L(e)$  of the coastline. As with method A, when  $e$  becomes smaller, the observed length  $L(e)$  increases without limit, and  $L(e)$  is related to the fractal dimension,  $D$  by Richardson’s empirical relation Eq. 1. This method is in fact very similar to method A and while  $L(e)$  is sensitive to the relationship between land and sea (considers both a lake and an island with identical shorelines) the fractal dimension is unaffected.

4. MANDELBROT’S METHOD C

Consider all the points with distances to the coastline of no more than  $e$ . These points form a strip of width  $2e$ , and the suggested length  $L(e)$  of the coast is the area of the strip divided by  $2e$ . This method is actually based upon a technique known as ‘Minkowski’s sausage logic’ which is one of a number of standard procedures used in mathematics to ‘tame’ wildly irregular curves. The procedure is known to date back at least to Minkowski 1901 or may be even older. The method is covered by Mandelbrot and is also briefly explained in Ch. 2 of Kaye’s book *A Random Walk through Fractal Dimensions*. Once again, as with the two previous methods, as  $e$  decreases  $L(e)$  increases, and both  $e$  and  $L(e)$  are related by Richardson’s Eq. (1).

5. MANDELBROT’S METHOD D

Cover the coastline with the minimal number of discs of radius  $e$ , not necessarily centered on the coastline as in C”.

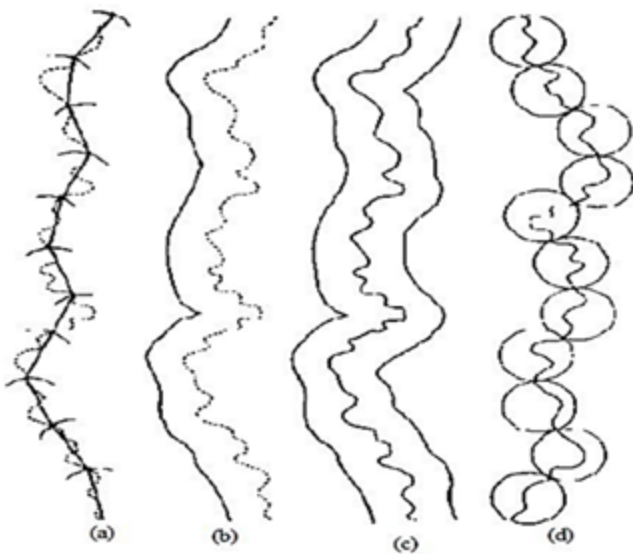


Fig 1: Mandelbrot’s four methods of estimating the length of a coastline.

2. MANDELBROT’S METHOD A

A yardstick of length  $e$ , we trace the yardstick along the coastline. The number of steps multiplied by  $e$  is an approximate length,  $L(e)$  of the coastline. For a coastline, when  $e$  becomes smaller, the observed length  $L(e)$  increases without limit. This is, in essence, the method used by Richardson who proposed the empirical relationship relating the length estimate,  $L(e)$  with the scale,  $e$  of measurement given by:



“Let  $L(e)$  be the total area of these discs divided by  $2e$ . This method is very similar to method C, but requires substantially less circles to construct. In fact it is also very similar to method A since a polygon linking the centres of every disc must approximate to the path followed by a yardstick of length  $e$ . As before, with the previous methods, of Mandelbrot, as  $C$  decreases  $L(e)$  increases and both  $e$  and  $L(e)$  are related by Richardson’s Eq 1.

## 6. DIFFERENT FRACTAL METHODS

### 1.1. STRUCTURED WALK METHOD (RICHARDSON) AND THE HAUSDORF DIMENSION

Kaye (1990) (also called caliper, divider, equi-spaced random walk or yardstick), developed a vector-based method based on the initial study of Richardson (1961). It states [9] that walking around the perimeter of an object with a pair of compasses (or divider) of a finite stride length. The perimeter of the outline is calculated by the number of steps needed to span the outline multiplied by the stride length ( $\lambda$ ). Any change in the stride length, which is equivalent to changing the scale of observation, will produce another estimate of the perimeter. By plotting the log of the perimeter against the log of the step length (Richardson plot) forms a linear relationship from which the fractal dimension ( $F_d$ ) is derived by the relationship,  $F_d = 1 - s$

Where  $s$  is the slope of the plot and 1 represents the topological dimension of a line.

### 1.2 BOX-COUNTING METHOD (BC)

This method [11] was defined by Russel et al. (1980), it is the most frequently used and most popular method. By covering a binary signal with boxes of length  $r$ , the FD is estimated as:

$$FD = -\lim_{r \rightarrow 0} \frac{\log(N(r))}{\log(r)} \quad (2)$$

where  $N(r)$  is number of boxes needed to completely cover the signal.

Normant and Tricot (1991) showed that this method is not theoretically well founded and is valid only for statistically self-similar signals.

Moreover, as the iteration for different sizes of  $r$  can produce various sizes of  $N(r)$ , the grid should be randomly relocated at each iteration (Appleby, 1996). More recently, Pruess (2007) showed that the computation of the FD is box size sensitive.

### 1.3. DIFFERENTIAL BOX-COUNTING METHOD (DBCM)

It was proposed [11] by Chaudhuri and Sarkar (1995) to solve some of limitations of the BM method. It has the advantage to work on grey-scale images and thus the binarization step is avoided. The signal is partitioned into boxes of various sizes  $r$  and  $N(r)$  is computed like the difference between the minimum and the maximum grey levels in the  $(i, j)$ th box. This step is repeated for all boxes and the FD is estimated.

### 1.4. EXTENDED COUNTING” METHOD (XCM)

The extended counting method [11] “XCM” (Sandau and Kurz, 1997) was proposed as an alternative to the BCM. The BCM is applied to many subsets of a fractal set and the maximum of the subsets’ dimensions is taken as the FD of the set. On the other hand, the BCM, as it is used for subsets, it is extremely simplified (a box-counting regression line is built only on the basis of two points). This method can be compared to the BCM because the FD is computed on binary signals. Although the BCM is the most widespread, Sandau and Kurz (1997) showed that the XCM presents some benefits. Indeed, XCM, in contrast to BCM, calculates a measure of complexity without regression. Hence, this measure grows monotonously with complexity and is determined by the most complex region of the signal. This corresponds to an important feature of FD, the maximum property, which is approximately fulfilled by XCM, but not by BCM. Also, XCM is less sensitive to the signal rotation and translation influence

### 1.5. FRACTIONAL BROWNIAN MOTION (FBM) METHODS

The fractal model [11] based on fBm is a non-stationary model



and is often used to describe random phenomenon. Pentland (1984) showed that most fractals encountered in physical models are fractal Brownian functions (fBfs). An fBf (Mandelbrot, 1975)  $f$  is a generalization of Brownian motion where the expected value of the intensity difference between two points is zero but where the square of the difference is proportional to the distance between the points at a power  $2H$ .

The FD of an  $n$ -dimension fBf is defined by:

$$FD = n + 1 - H \quad (3)$$

Fractal Brownian functions are statistically self-affine (Mandelbrot, 1983). It follows that linear transformations and scaling of afBf do not affect its FD. With this formalism, the FD of a fractal Brownian function is invariant with transformations.

#### I.6. VARIOGRAM METHOD

The variogram method [11] is based on the statistical Gaussian modelling of images. This method attempts to solve the inverse problem: given an image, the FD is estimated by assuming that it is derived from a fractional Brownian motion (Goodchild, 1980).

This algorithm provides robust estimations of the FD (Soille and Rivest, 1996). Indeed the advantages of the variogram method are its applicability to irregularly distributed data sets and to surfaces with an underlying trend, as commonly occur in topography. For surfaces with a trend of higher than linear order, the residual variogram is preferable. Although the theoretical derivation assumes second-order stationarity, the surface does not need to be stationary to use the variogram method. However it was shown that dividing a signal into an insufficient number of clusters makes the variogram method unable to estimate the FD, but when a sufficient large number of clusters is used it is possible to detect a very sharp drop toward the correct value, followed by slow convergence (Kolibal and Monde, 1998).

On the other hand, the variogram method yields accurate results only for low dimensions surfaces. For higher

dimensions surfaces, the method is unstable (Lam et al., 2002).

#### I.7. THE POWER SPECTRUM

Power spectrum method [11] (Pentland, 1984) is based on the power spectrum dependence of fractional Brownian motion. In this method, each image line is Fourier transformed, the power spectrum is evaluated and then all these power spectra are averaged. FD is computed as the slope.

Asvestas et al. (1998) defined a modified version to estimate the FD of a two variable fBm functions from its average power spectrum. The method is called Power Differentiation method. The authors showed that their method is more robust in the presence of white noise. The main drawback is that the method is efficient only with surfaces exhibiting an exponential power spectrum.

#### I.8. AREA MEASUREMENT METHODS

Area measurement methods [11] use structuring elements (triangle, erosion, dilatation...) of various scales  $r$  and compute the area  $A(r)$  of the signal intensity surface at scale  $r$ . The FD is obtained by the slope of the best fitting line at the points  $(\log(r), \log(A(r)))$ . In this methods class, three algorithms are the most used.

#### I.9. ISARITHM METHOD (IM)

The idea of the isarithm method [11] (Shelberg et al., 1983) is to define the complexity of isarithm or contour lines, needed to approximate the complexity of a surface. This method is only defined for the 2D case. A series of isarithms (e.g., contours) based on the data values are formed on the image. The FD of each isarithm can be estimated with the walking divider method and the FD of the image is the average FD of the isarithms plus one.

Shelberg et al. (1983) showed that the method can be used to estimate the FD for the non-self-similar surfaces. Klinkenberg and Goodchild (1992) found results with this method to be poor. Clarke (1986) criticized the isarithm method because the resulting dimension was likely to depend on the values of the isarithms and isarithm interval.



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#### I.10. BLANKET METHOD (BM)

The blanket algorithm [11] was originally devised by Peleg et al. (1984) in order to calculate the area of a grey level surface and thus the FD of a 3D structure. The algorithm is based upon Mandelbrot's method and ultimately upon Minkowski's sausage logic. In the algorithm, Peleg et al. considered all the points in 3D space at a distance  $e$  from the surface, covering the surface with a "blanket" of thickness  $2e$ . This blanket is defined by two surfaces, an upper surface and a lower surface (defined by dilatation and erosion of the image).

One of the advantages of the method is its robustness against gray levels changes. Another benefit is that the use of asymmetric structuring elements allowed the identification of anisotropic structures within the image (Chappard et al., 2001). Asvestas et al. (1998) showed that the BM is efficient only when the theoretical value of the FD is relatively low.

#### I.11. TRIANGULAR PRISM METHOD (TPM)

The triangular prism method [11] compares the surface areas of triangular prisms with the pixels area (step size squared) in log-log form (Clarke, 1986). The method derives a relationship between the surface area of triangular prisms defined by the grey-level values of the image and the step size of the grid used to measure the prism surface area.

Moreover the TPM was also found to be sensitive to noise or extreme grey-level values. However, beyond these limits, the method is the fastest and gives more accurate results than the BM and IM methods (Kolibal and Monde, 1998). The accuracy and efficiency of the three methods were evaluated on images from the Cantor set. In 2006 Sun (2006) proposed three new methods to compute the fractal dimension based on Clarke's TPM method, called the max-difference method, the mean-difference method and the eight-pixel method. Results showed that the methods are more robust than the Clarke's method for synthetic images with complex textures.

## II. APPLICATIONS

### I.1. FRACTAL OF ENGINEERING CERAMICS

To characterize the engineering ceramics ground surface [12] Fractal method is used. Fractal method provides parameters which are independent of sampling length and resolution of the measuring instrument, and can reflect intrinsic property of surface. This method shows the fractal dimension effect on the texture of engineering ceramics. As texture in engineering ceramics surface is fine and dense, when the fractal dimension is high; the texture is coarse and sparse, when the fractal dimension is low. When the value of fractal dimension increases, the material ratio of the profile  $Pmr(c)$  of various materials in general also increase. With the higher value of the fractal dimensions, they can show the larger contact area and the better wear resistance for the same material. Meanwhile, the size order of fractal dimension is steel, zirconia, silicon nitride and alumina. It is the same as the order of fracture toughness of four materials.

### 2.2 GEAR FAULT DIAGNOSIS

In Gear fault diagnosis [15] two methods, as fusion method of local mean decomposition (LMD) and generalized morphological fractal dimensions (GMFDs) is proposed. The GMFDs are extracted from signals of different lengths, the steps in this diagnosis procedure ones sampled under three different working conditions of load and speed, ones without decomposition of LMD. The test of gear signals are held and verified and it shows that the proposed method is superior and can diagnose gear faults accurately.

### 2.3 NANOPARTICLES

The transmission electron micrographs (TEM) of the Pt-Cu bimetallic nano-particles [17] at different weight ratios, stabilized by PVP (polyvinyl pyrrolidone), obtained Fractal dimensions. This indicates a bimodal fractal behaviour characterized by a higher fractal dimension for higher self-similarity domains and a small fractal dimension at low scales. This is due to a mixed-fractal structure, rather than a lack in instrument resolution. It also shows that both box-counting



fractal dimension and information dimension increase as concentration of Cu increases. A method from fractal analysis is developed to compute the distribution of the number of particles versus the radius; the direct analysis of the images is in good agreement with results.

### III. CONCLUSION

Fractal geometry provide mathematical tool for identify the mechanisms behind the generation of wear particles. Boundary fractal dimension (FD) finds the surface texture, shape and size of a particle for identify the causes of wear. fractal geometry applied in imaging application for characterization as by SEM images. This method has significance and advantages than the previous method as Ferrography. Large numbers of fractal analysis methods developed over the last years. The best fractal method is that which is perfect in computational techniques and provide sufficient accuracy. Fractal analysis evaluate fractal dimension (FD) which allows wide description of image of particles. The EXACT algorithm provides highly accurate and detailed Richardson plots so this is called versatile but it takes more time for analysis, HYBRID and FAENA algorithm preferred for fast analysis. The FAST analysis is fastest of the techniques but least accurate. The box-counting method is also most widely used, despite its drawbacks (Binarization of the signal, construction of empty boxes, grid effect, etc.). Important issue in the computation of FD is choice of methods. Limitation in the fractal analysis is to describe objects by a single fractal value whereas they exhibit a multifractal behaviour. As conclusion fractal geometry is a powerful tool to deal with problems in image analysis. The dimensionality of images and the suitable algorithm are certain terms are taken into consideration for fractal and multifractal analysis.

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